Inexact Proximal Point Algorithms for Inclusion Problems on Hadamard Manifolds

Feeroz Babu *

Abstract

This paper proposes inexact proximal point algorithms for computing the zeros of sum of two set-valued vector fields on Hadamard manifolds. The convergence results of the proposed algorithm are established under the assumption that the one set-valued vector field is monotone and lower semicontinuous and another one is monotone and upper Kuratowski semicontinuous. An example is given to illustrate the proposed algorithms and convergence result. As an application of the proposed algorithm and convergence result, an algorithm and its convergence result are derived for solving set-valued variational inequality problems in the setting of Hadamard manifolds.

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A partitioning method for linear and/or linear fractional rank-two programs

Riccardo Cambini *

Abstract

Low rank problems are nothing but nonlinear minimization problems over polyhedrons where a transformation of the variables provides an objective function which actually depends on very few variables. These problems are often used in in applications, for example in concave quadratic minimization problems, multiobjective/bicriteria programs, location-allocation models, quantitative management science, data envelopment analysis, efficiency analysis and performance measurement. The aim of this talk is to deepen on the study of a solution method, based on a partitioning approach, for some classes of nonconvex problems having a polyhedral feasible region expressed by means of inequality and/or box constraints. The considered objective function is either of the kind $\phi(c^T x + c_0, d^T x + d_0)$ or of the kind $\phi(c^T x + c_0, \frac{d^T x + d_0}{h^T x + h_0})$.

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Convergence to Nash equilibria in ratio-bounded games

Francesco Caruso*, Maria Carmela Ceparano, Jacqueline Morgan

Abstract

In a two-player non-cooperative game framework, we deal with the affine relaxations of the so-called *best response algorithm*, that is an iterative scheme where the updated strategy of player i is obtained by selecting a minimizer of player i's payoff function given the strategy of the other player coming from the previous step. In order to be able to specify the convergence of any type of affine relaxation of the best response algorithm, we define a new class of convex games, called *ratio-bounded games*. This class contains games broadly used in literature (such as weighted potential and zero-sum games), both in finite and in infinite dimensional setting. Its definition relies on a unifying property and on three associate key-parameters explicitly related to the data. Depending on how the parameters are ordered, we provide a classification of the ratio-bounded games in four subclasses such that, for each of them, the following issues are answered when the strategy sets are real Hilbert spaces: existence and uniqueness of the Nash equilibria, global convergence of the affine relaxations of the best response algorithm, estimation of the related errors, determination of the algorithm with the highest speed of convergence and comparison with the known results.

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On a functional inequality of Alzer and Salinas

Włodzimierz Fechner *

Abstract

The results of the talk are motivated by a paper by Alzer and Salinas [1]. They study the following functional inequality:

$$f(x)f(y) - f(xy) \leq f(x) + f(y) - f(x+y) \tag{1}$$

for mapping $f\colon \mathbb{R} \to \mathbb{R}$. The main theorem of [1] says that if f is differentiable at zero and convex or concave, then, either f is constant or it is equal to the identity mapping. An open problem to determine all solutions of (1) under more flexible assumptions was formulated in [1]. During the talk, we will make an approach to answer it and we will present two results on inequality (1).

Results of the talk are based on paper [2].

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Computer assisted investigations connected to m-convexity of sets

Attila Gilányi [*]

Abstract

According to a definition of Gheorghe Toader [1], if $m \in [0,1]$ is a fixed real number, a set $H \subseteq \mathbb{R}^2$ is called m-convex if $tx + m(1-t)y \in H$ for all $x, y \in H$ and $t \in [0,1]$. The m-convex hull of a nonempty set $S \subseteq \mathbb{R}^2$ is defined as the intersection of all m-convex subsets of $\mathbb{R}^2$ containing S. Connected to these concepts, we present a computer program developed in the computer algebra system Maple, which determines the m-convex hulls of sets consisting of finitely many points in the plane. (Joint work with Roy Quintero and Lan Nhi To.)

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Relaxed-inertial proximal point algorithms for problems involving strongly quasiconvex functions

Sorin-Mihai Grad*

Abstract

Introduced in in the 1970's by Martinet for minimizing convex functions and extended shortly afterwards by Rockafellar towards monotone inclusion problems, the proximal point algorithm turned out to be a viable computational method for solving various classes of optimization problems, in particular with nonconvex objective functions.

We propose a relaxed-inertial proximal point type algorithm for solving optimization problems consisting in minimizing strongly quasiconvex functions whose variables lie in finitely dimensional linear subspaces, that can be extended to equilibrium problems involving such functions. We also discuss possible modifications of the hypotheses in order to deal with quasiconvex functions. Numerical experiments confirm the theoretical results, in particular that the relaxed-inertial algorithms outperform their "pure" proximal point counterparts [3, 4].

This talk is based on joint work [1, 2] with Felipe Lara and Raúl Tintaya Marcavillaca (Universidad de Tarapacá).

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The role of nonlinear scalarization functions in characterizing generalized convex vector functions

Christian Günther [*]

Abstract

The aim of this talk is to present characterizations of cone-convex and explicitly cone-quasiconvex vector functions with respect to a proper closed solid convex cone of a real linear topological space. These characterizations are given in terms of classical convexity and explicit quasiconvexity of certain real-valued functions, defined by composing the vector-valued function with the nonlinear scalarization function introduced by Gerstewitz (Tammer) in 1983.

This talk is based on joint works with Nicolae Popovici.

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Existence results for weak efficient solutions of vector optimization problems via scalarization

C. Gutiérrez *　　R. López †

Abstract

The aim of the talk is to show some recent existence results for weak efficient solutions of vector optimization problems in several settings, which cover the coercive, noncoercive and Weierstrass cases. They are obtained by considering the concept of colevel set and a scalarization approach based on a version of the so-called Gerstewitz functional.

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Quasiconvex families of functions and quasimonotone families of operators

N. Hadjisavvas (with F. Flores-Bazán and Y. García) *

Abstract

It is well-known that the sum of two quasiconvex functions is not necessarily quasiconvex. Likewise, the sum of two quasimonotone operators is not quasimonotone in general. We introduce the notions of quasiconvex family of functions, and quasimonotone family of operators. Such families are shown to be stable by addition. We give characterizations of a family of functions to be quasiconvex, and the relation to quasimonotonicity of the family of their subdifferentials. We examine when other properties, such as strict or semistrict quasimonotonicity, are also preserved by addition.

Some particular attention is given to properly quasimonotone operators, because of their importance. We especially describe the case when a quasimonotone operator is not properly quasimonotone.

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Pseudoaffine variational inequalities and pseudolinear programming problems

Miruna Beldiman *, Andrei Dan Halanay †

Abstract

We find equivalence conditions between a variational inequality and a generalized pseudolinear programming problem, under generalized differentiability and pseudoaffinity hypothesis.

In order to obtain this, first we will establish some properties of bifunctions on generalized invex sets. Then we introduce notions of generalized convexity for a function (with respect to a bifunction) and after that we make the connection between this convexity and generalized monotonicity.

Follows the statement of the main problems and the characterization of their respective sets of solutions. Finally we give a relation between these two sets.

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Duality and optimality conditions in convex robust optimization

Abderrahim Hantoute *

Abstract

We provide new optimality conditions for convex robust optimization problems in an infinite-dimensional framework, which are both exact and fuzzy. While the fuzzy calculus only requires the lower semi-continuity of the given data functions, the exact conditions involve continuity type criteria which extend the usual constrained qualification of ordinary convex optimization, namely the classic qualification condition of Fenchel-Moreau-Rockafellar. These results and the underlying structure suggest the introduction of different dual optimization models which, compared to the usual models of ordinary convex optimization, involve certain additional terms related to the effective domain of the objective function.

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An augmented subgradient algorithm for minimizing nonsmooth DC functions

Najmeh Hoseini Monjezi *

Abstract

In this talk we consider an unconstrained DC optimization problem

$$\begin{cases} \text{minimize} & f(x) \\ \text{subject to} & x \in \mathbb{R}^n, \end{cases} \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}$, $f(x) = f_1(x) - f_2(x)$ and functions $f_1, f_2 : \mathbb{R}^n \to \mathbb{R}$ are convex and in general, nonsmooth. Some important practical problems can be formulated as a nonsmooth DC program of the form (1). They include supervised data classification, cluster analysis, clusterwise linear regression analysis and edge detection problems; for more information see [1] and references therein.

To date, several methods have been developed to locally solve Problem (1). They include, but not limited to the aggregate subgradient method [2], the proximal bundle method with the concave affine model [4], the proximal bundle method for DC optimization [3] and the double bundle method [5].

We propose an augmented subgradient algorithm to solve problem (1) [1]. At each iteration, by using several subgradients of the first DC component and one subgradient of the second DC component of the objective function, a search direction is calculated. If the direction is descent, then an Armijo-type line search procedure to find the next iteration point is applied. Otherwise (the direction is not descent) by using new information of the both components of the objective function, a new search direction is computed and the algorithm is repeated. The global convergence is proved in the sense that, the sequence of points generated by the method converges to a critical point of the unconstrained DC optimization problem. At the end, in order to study the reliability and efficiency of the proposed algorithm, Fortran implementation of it is prepared and numerical experiments have been made by using some academic test problems with nonsmooth DC objective functions. In addition, the performance of the algorithm is compared with that of two general nonsmooth optimization solvers and five solvers specifically designed for unconstrained DC optimization. Computational results show that the developed method is efficient and robust for solving nonsmooth DC optimization problems.

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Recent results in Asysmptotic Analysis with Applications in Optimization Theory

Alfredo Iusem *

Abstract

We present some recent results on Asymptotic analysis with applications to Continuous Optimization. After a short exposition of the classical results regarding recession cones and asymptotic functions in the convex case, as well as more recent extensions to more general settings, e.g. quasiconvex functions, we focus on three recent developments:

1) Applications of the q-asymptotic function to optimization and equilibrium problems in Banach spaces, establishing results on non-emptiness and compactness of the solution set.

2) Applications of the classical asymptotic function to Mixed Variational Inequality Problems in finite dimensional spaces, obtaining results of the same kind.

3) Introduction of a new second order asymptotic function, which provides information on the convexity (concavity) of the original function at infinity.

These results have been proved in [1], [2], [3] and [4], coauthored by Felipe Lara Obreque.

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Exponential Penalty Approach for Multi-dimensional Vvariational Problem with Inequality Constraints

Anurag Jayswal, Ayushi Baranwal *

Abstract

In this article, we describe a method to deal with a multi-dimensional variational problem with inequality constraints using an exponential penalty function. By utilizing the exponential penalty function, we formulate an unconstrained multi-dimensional variational problem and examine the relationships between the optimal solution to the considered multi-dimensional variational problem and the sequence of minimizers of the unconstrained multi-dimensional variational problem. The convergence of the proposed exponential penalty approach is also investigated, which shows that a convergent subsequence of the sequence of minimizers of the unconstrained multi-dimensional variational problem approaches an optimal solution to the multi-dimensional variational problem. Further, an illustrative application (to minimize a manufacturing cost functional of a production firm) is also presented to confirm the effectiveness of the proposed outcomes.

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Semi-quasidifferentiability in quasiconvex nonsmooth optimization

A. Kabgani[1] *, M. Soleimani-damaneh[2]

Abstract

Semi-quasidifferential notion, as a new tool to deal with nonsmooth optimization problems, is exploited to provide some Karush–Kuhn–Tucker type optimality conditions for quasiconvex programming. We obtain some necessary and sufficient optimality conditions for a constrained nonsmooth quasiconvex optimization problem in terms of semi-quasidifferentials. Moreover, we show that a broad class of functions, including Gâteaux differentiable functions, convex and concave functions, and Clarke's regular functions, are semi-quasidifferentiable. Regarding this, our outcomes generalize several related results existing in the literature.

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On quasidifferentiable mathematical programs with equilibrium constraints

Vivek Laha *& Harsh Narayan Singh†

Abstract

The aim of this paper is to study mathematical programs with equilibrium constraints [2] involving quasidifferentiable functions [1] and to synthesize suitable optimality conditions. We derive Fritz-John (FJ) and Karush-Kuhn-Tucker (KKT) type necessary optimality conditions at an optimal point in the framework of the quasidifferentiable analysis. Further, we prove several sufficient optimality conditions for a stationary point to be an optimal solution of the quasidifferentiable mathematical program with equilibrium constraints under suitable choice of generalized convex functions.

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On redistributing algorithms and Shannon's Entropy

Flavia-Corina Mitroi-Symeonidis*, Eleutherius Symeonidis†

Abstract

The mathematical problems discussed in our talk arise in the framework of time series analysis. The permutation entropy is used, and the underlying probabilities are established using redistributing algorithms. We point out the changes of Shannon's entropy due to modifications of the distribution probability by means of redistributing algorithms.

Keywords: permutation entropy; time series analysis; redistributing algorithms

Mathematics Subject Classification: Primary 94A17; Secondary 37M10, 37A35

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Generalized Newton Methods via Variational Analysis

Boris Mordukhovich *

Abstract

In this talk we present novel locally and globally convergent Newton-type methods to solve unconstrained and constrained problems of nonsmooth optimization by using tools of variational analysis and generalized differentiation. These methods are coderivative-based and employ generalized Hessians (coderivatives of subgradient mappings) associated with problems of convex composite optimization, where one of the terms may be extended-real-valued. The proposed globally convergent algorithms are of two types. The first one extends the damped Newton method and requires positive-definiteness of the generalized Hessians for its well-posedness and efficient performance, while the other algorithm is of the Levenberg-Marquardt type being well-defined when the generalized Hessians are merely positive-semidefinite. The obtained convergence rates for both methods are at least linear, but becomes superlinear under the so-called semismooth* property of subgradient mappings. Problems of convex composite optimization are investigated with and without the strong convexity assumption on of smooth parts of objective functions by implementing the machinery of forward-backward envelopes. Numerical experiments are conducted for a basic class of Lasso problems by providing performance comparisons of the new algorithms with some other first-order and second-order methods that are highly recognized in nonsmooth optimization.
This talk is based on joint work with P. D. Khanh, T. D. Phat, M. E. Sarabi and D. B. Tran.

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The betweenness property

Adriana Nicolae *

Abstract

In this talk we discuss convexity-like properties formulated in terms of betweenness relations and their relevance with other geometric conditions. As a framework, we consider geodesic metric spaces and, in particular, normed spaces. We also focus on the role of these properties in the study of various problems such as the asymptotic behavior of iterates of classes of mappings or a pursuit-evasion game.

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Ohlin-type result for strongly convex functions and set-valued maps

Kazimierz Nikodem *

Abstract

Let $I \subset \mathbb{R}$ be an interval. A function $f : I \to \mathbb{R}$ is called *strongly convex with modulus* $c > 0$ if

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) - ct(1-t)(x-y)^2$$

for all $x, y \in I$ and $t \in (0,1)$. Let $(\Omega, \mathcal{A}, P)$ be a probability space. Given a random variable $X : \Omega \to \mathbb{R}$ we denote by F_X, $\mathbb{E}[X]$ and $\mathbb{D}^2[X]$ the distribution function, the expectation and the variance of X, respectively. The following Ohlin-type result for strongly convex functions is presented:

Theorem. *Let $X, Y : \Omega \to I$ be square integrable random variables such that $\mathbb{E}[X] = \mathbb{E}[Y]$. If there exists $t_0 \in \mathbb{R}$ such that*

$$F_X(t) \leq F_Y(t) \;\; if \;\; t < t_0 \quad and \quad F_X(t) \geq F_Y(t) \;\; if \;\; t > t_0,$$

then

$$\mathbb{E}[f(X)] - c\mathbb{D}^2[X] \leq \mathbb{E}[f(Y)] - c\mathbb{D}^2[Y]$$

for every function $f : I \to \mathbb{R}$ strongly convex with modulus c.

As an application various inequalities related to strongly convex functions are obtained in a simple unified way. Finally, counterparts of the Ohlin theorem for convex and strongly convex set-valued maps are given.

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Characterization of segment and convexity preserving maps

Zsolt Páles *

Abstract

Let X and Y be (real) linear spaces. For $a, b \in X$ (or $a, b \in Y$) the closed segment $[a, b]$ and the open segment $]a, b[$ connecting the points a and b are defined by

$$[a, b] := \{ta+(1-t)b \mid 0 \le t \le 1\}, \qquad]a, b[:= \{ta+(1-t)b \mid 0 < t < 1\}.$$

Given a convex subset $D \subset X$ and a map $f : D \to Y$, we can consider two *convexity preserving properties* for f. We say that f preserves convexity if $f(K)$ is convex for all convex subset $K \subseteq D$. Analogously, we say that f^{-1} preserves convexity or f is inversely convexity preserving if $f^{-1}(K)$ is convex whenever K is a convex subset of $f(D)$. It is immediate to see that f is convexity preserving if and only if

$$[f(x), f(y)] \subseteq f([x, y]) \qquad (x, y \in D).$$

On the other hand, f is inversely convexity preserving if and only if

$$[f(x), f(y)] \supseteq f([x, y]) \qquad (x, y \in D).$$

Functions enjoying both of the above properties, i.e., satisfying

$$f([x, y]) = [f(x), f(y)] \qquad (x, y \in D),$$

are called *segment preserving maps*. Therefore, f is segment preserving if and only if it is convexity and also inversely convexity preserving.

If the closed segments are replaced by open segments in the above definitions then we speak about *strict* convexity and segment preserving properties for f. Clearly, strict convexity or segment preserving maps are always convexity or segment preserving (in the corresponding sense), the converse, however, may not be valid.

The obvious examples for (strict) convexity and segment preserving maps are *affine maps*, i.e., functions of the form $f(x) = A(x)+a$, where $A : X \to$

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Y is linear and a is a constant vector. It is a natural question if there exist other types of segment preserving functions.

One can trivially see that if X and Y are equal to the set of real numbers, then $f : X \to Y$ is segment preserving if and only if it is continuous and either increasing or decreasing; furthermore, f is strictly segment preserving if and only if it is continuous and either strictly increasing, or strictly decreasing, or constant. Therefore, for the first sight, the class of such maps seems to be even more complicated in the higher-dimensional setting. However, as our main result shows, if the range of f is at least two dimensional, then the description is easier: strict segment preserving maps, moreover strict inversely convexity preserving maps can always be expressed as the ratio of an Y-valued and a real-valued affine map.

Computing the Pareto front in Multiobjective Linear Mixed Integer Fractional Programming

João Paulo Costa and Maria João Alves *

Abstract

This communication presents a Branch Bound technique to compute all the nondominated solutions in multiobjective linear mixed integer fractional programming (MOLMIFP). The technique provides not only supported but also unsupported nondominated solutions, that is, those that are dominated by unfeasible convex combinations of other nondominated solutions. Note that nondominated solutions associated with a weight vector are always supported. It can be observed that the nondominated solution set of a MOLMIFP problem has, in general, a significant part of unsupported solutions.
The difference between the computed nondominated solutions, supported or unsupported, can be controlled by a pre-defined step. Several computational performance tests will be presented

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Coderivative Characterizations of Quasiconvexity and Pseudoconvexity

Vo Thanh Phat*

Abstract

For a $\mathcal{C}^2$-smooth function on a finite-dimensional space, a necessary condition for its quasiconvexity is the positive semidefiniteness of its Hessian matrix on the subspace orthogonal to its gradient, whereas a sufficient condition for its strict pseudoconvexity is the positive definiteness of its Hessian matrix on the subspace orthogonal to its gradient. Our aim in this talk is to extend those conditions for $\mathcal{C}^{1,1}$-smooth functions via Mordukhovich coderivatives. (This talk is based on the joint work with **Pham Duy Khanh**[1])

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On optimality and duality results for multi-objective E-variational problems and application to cake eating problem

Nisha Pokharna Indira P.Tripathi *

Abstract

In this paper, the concept of E-convexity is introduced as an extension of convexity defined for variational optimization problems. The class of multi-objective variational problems in which the functions involved are (not necessarily differentiable) E-differentiable has been studied in this paper and to solve this problem corresponding multi-objective E-variational problem is constructed using the E operator. Under the hypothesis of E-convexity, the necessary and sufficient optimality conditions for the multi-objective E-variational programming problem are derived. The Mond-Weir dual problem has been formulated for the multi-objective E-variational problem, and duality results have been obtained under the E-convexity.Furthermore, we have studied a cake eating problem, and the solution to this problem is obtained using the sufficiency theorem, to highlight the importance of the results developed in this paper. Non-trivial examples are also included in the paper at appropriate places to support the findings.

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A Single Leader Radner Equilibrium problem: industrial symbiosis in an Eco-Industrial Park

Rossana Riccardi [*], Elisabetta Allevi [†] Didier Aussel [‡] Domenico Scopelliti [§]

Abstract

In this study, we propose a bilevel programming model for a theoretical industrial symbiosis network located in an Eco-Industrial Park (EIP). At the upper level, the leader can be interpreted as an authority that is in charge of the ecological concerns, while, at the lower level, a finite number of enterprises act as followers with economic objectives. Within the considered system and given the followers demand, the leader wants to minimize the consumption of the natural resources and the social cost arising for the community. Based on the EIP authority decisions, all enterprises compete with each other in a parametric non-cooperative game with the strategies of the EIP authority as exogenous parameters. The game evolves in a sequence of finite future periods, so that uncertainty on resource fluxes and environmental conditions has to be taken into account. This uncertainty is expressed through a finite set of all possible situations that can occur at each period. In this framework, then, a sequential game in which the enterprises controls, trades, and consumes a finite number of different natural resources is settled. The lower-level problem is formulated as a Radner equilibrium problem, and we aim to investigate on the connection between the leader behavior and the equilibrium problem of the followers.

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Remarks on Fixed Point and Equilibrium Problems

Wilfredo Sosa*

Abstract

In this manuscript we consider the Fixed Point Problem for two kinds of mapping, one of them are maximal σ premonotone mappings. The idea is transform the Fixed Point Problem onto an Equilibrium Problem in the sense of Blum-Oettli. Here, we introduce two ways in order to do it. The first one explore the properties of maximal σ premonotone mappings and the second one is for any mapping, non necessarily σ premonotone. Also we establish a methodology in order to extend any σ premonotone mapping to a maximal σ premonotone mapping.

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Some remarks on operators on the spaces of smooth functions related to derivatives and entropy

Aleksandra Świątczak *

Abstract

During the talk, we will discuss certain operator equations and inequalities in spaces of smooth functions. Our research involves applications of some tools of functional inequalities and convex functions. Our starting point is recent research connected with operator relations characterizing derivatives that were mainly carried out by H. KÃűnig and V. Milman and are mostly collected in their monograph [1]. We will put an emphasis on connections with various notions of generalized convexity and monotonicity.

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Relationships between constrained and unconstrained multi-objective optimization and application in location theory

Christiane Tammer, Christian Günther *

Abstract

The main purpose of the lecture is to investigate relationships between constrained and unconstrained multi-objective optimization problems, see [1, 2]. We mainly focus on generalized convex multi-objective optimization problems, i.e., the objective function is a componentwise generalized convex (e.g., quasi-convex, semi-strictly quasi-convex or explicitly quasi-convex) function and the feasible domain is a not necessarily convex set. Beside the field of location theory, the assumptions of generalized convexity are found in several branches of Economics, e.g., in the field of utility theory.

In the talk, we formulate the basic constrained multi-objective optimization problem and the corresponding extended unconstrained one, we introduce solution concepts and recall results about generalized convex and semi-continuous functions.

Under suitable assumptions (e.g., generalized convexity assumptions), we derive a characterization of the set of (weakly) efficient solutions of a constrained multi-objective optimization problem using characterizations of the set of (weakly) efficient points of unconstrained multi-objective optimization problems. Furthermore, we present an assertion that provides lower and upper bounds for the sets of (weakly) efficient solutions for multi-objective optimization problems involving nonconvex constraints.

Moreover, we deduce necessary optimality conditions using our results concerning relationships between constrained and unconstrained multi-objective optimization problems. We apply these optimality conditions for deriving a proximal-point-algorithm for solving constrained multi-objective location problems, see [3].

Our results can be applied to constrained point-objective location problems involving mixed gauges defined by

$$\begin{pmatrix} \eta_1(x-a^1) \\ \dots \\ \eta_m(x-a^m) \end{pmatrix} \to \min_{x \in X},$$

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where $\eta_1, \dots, \eta_m : \mathbb{R}^n \to \mathbb{R}$ represent special distance functions (gauges) and $a^1, \dots, a^m$ are finitely many given points in $\mathbb{R}^n$. The MATLAB-based software tool

Facility **L**ocation **O**ptimizer (**FLO**)

can be used for solving special types of single- as well as multi-objective location problems involving different distances measures. For more information, see http://www.project-flo.de.

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Inexact reduced gradient methods in smooth nonconvex optimization

Dat Ba Tran *

Abstract

The talk introduces new gradient-type methods with inexact gradient information for finding stationary points of nonconvex continuously differentiable functions on finite-dimensional spaces. A general scheme for inexact reduced gradient (IRG) methods with different stepsize selections is proposed to construct sequences of iterates with stationary accumulation points. Convergence results with convergence rates for the developed IRG methods are established under the Kurdyka-Lojasiewicz property. The conducted numerical experiments confirm the efficiency of the proposed algorithms.

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Common solutions to a finite family of inclusion problems and an infinite family of fixed point problems by a generalized viscosity implicit scheme including applications

Rajat Vaish [*]

Abstract

This manuscript deals with two problems: the first one is a variational inclusion problem involving an m-accretive mapping and a finite family of inverse strongly accretive mappings, and the other one is a fixed point problem having an infinite family of strict pseudo-contraction mappings in Banach spaces. To approximate the common solution of these problems, we design a generalized viscosity implicit iterative scheme with Meir–Keeler contraction. A strong convergence result for the proposed iterative scheme is established. Applications based on convex minimization problem, linear inverse problem, variational inequality problem and equilibrium problem are derived from the main result. The numerical applicability of the main result and some applications are demonstrated by three examples. Our result extends, generalizes and unifies the previously known results given in literature.

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Duality for sets of strong Slater points

Margarita M.L. Rodríguez & **José Vicente-Pérez**[*]

Abstract

The strong Slater condition plays a significant role in the stability analysis of linear semi-infinite inequality systems. This piece of work studies the set of strong Slater points, whose non-emptiness guarantees the fullfilment of the strong Slater condition. Given a linear inequality system, we firstly establish some basic properties of the set of strong Slater points. Then, we derive dual characterizations for this set in terms of the data of the system, following similar characterizacions provided also for the set of Slater points and the solution set of the given system, which are based on the polarity operators for evenly convex and closed convex sets. Finally, we present two geometric interpretations and apply our results to analyze the strict inequality systems defined by lower semicontinuous convex functions.

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Locally Maximal Monotone Operators and Applications

Khoa V.H. Vu *

Abstract

The talk discusses two notions of locally maximal monotone set-valued mappings on Hilbert spaces (both strong and non-strong) and their relationships. Using the Fréchet and mixed limiting coderivative, we establish full characterizations of the mentioned local concepts. Moreover, their equivalence is validated. From the obtained characterizations, a sum rule for locally maximal monotone operators are built. The talk is based on the joint work with Boris S. Mordukhovich (Wayne State University, Michigan, USA), Pham Duy Khanh (Ho Chi Minh city University of Education, Vietnam) and Vo Thanh Phat (Wayne State University, Michigan, USA).

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